

Exercise 50

Find the limit.

$$\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 + x - 2}$$

Solution

Rewrite the limit.

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 + x - 2} &= \lim_{x \rightarrow 1} \frac{\sin(x-1)}{(x+2)(x-1)} = \lim_{x \rightarrow 1} \frac{1}{x+2} \cdot \frac{\sin(x-1)}{x-1} \\ &= \left(\lim_{x \rightarrow 1} \frac{1}{x+2} \right) \left[\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x-1} \right] \\ &= \left(\lim_{x \rightarrow 1} \frac{1}{x+2} \right) \left(\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \right) \\ &= \left(\frac{1}{3} \right) (1) \\ &= \frac{1}{3} \end{aligned}$$